2.15. Let $V_N \equiv 0$. But, $V_{N'N} \neq 0$ because the system is not balanced,

However, KCL says $\tilde{I}_a + \tilde{I}_b + \tilde{I}_c = 0$,

Write some equations —

$$\tilde{I}_a = \frac{\tilde{E}_{aN} - \tilde{V}_{N'N}}{Z}, \quad \tilde{I}_b = \frac{\tilde{E}_{bN} - \tilde{V}_{N'N}}{Z}, \quad \tilde{I}_c = \frac{\tilde{E}_{cN} - V_{N'N}}{Z}$$

$\tilde{E}_{aN}$, $\tilde{E}_{bN}$, $\tilde{E}_{cN}$ given

Add the three equations (noting that $\tilde{I}_a + \tilde{I}_b + \tilde{I}_c = 0$)

$$\frac{\tilde{E}_{aN} - \tilde{V}_{N'N}}{Z} + \frac{\tilde{E}_{bN} - \tilde{V}_{N'N}}{Z} + \frac{\tilde{E}_{cN} - \tilde{V}_{N'N}}{Z} = 0$$

Multiplying by Z, and separating terms

$$3\tilde{V}_{N'N} = \tilde{E}_{aN} + \tilde{E}_{bN} + \tilde{E}_{cN}$$

$$\tilde{V}_{N'N} = \frac{\sqrt{2}\angle 45 + 1\angle -90 + 1\angle 180}{3} = \frac{\cancel{1} + \cancel{j1} - \cancel{j1} - \cancel{1}}{3} = 0$$

Coincidence!

So, using $-\tilde{E}_{aN} + \tilde{I}_a Z + \cancel{V_{N'N}}^{0} = 0$,

we get $Z = \dfrac{\tilde{E}_{aN}}{\tilde{I}_a} = \dfrac{\sqrt{2}\angle 45}{1\angle -10} = \sqrt{2}\angle 55^\circ$ A

Then, $\tilde{I}_b = \dfrac{E_{bN} - \cancel{V_{N'N}}^{0}}{Z = \sqrt{2}\angle 55^\circ} = \dfrac{1\angle -90^\circ}{\sqrt{2}\angle 55^\circ} = \dfrac{1}{\sqrt{2}}\angle -145^\circ$ A

$$\tilde{I}_c = \frac{\tilde{E}_{cN} - \cancel{\tilde{V}_{N'N}}^{0}}{Z = \sqrt{2}\angle 55^\circ} = \frac{-1}{\sqrt{2}\angle 55} = \frac{1}{\sqrt{2}}\angle 125^\circ \text{ A}$$

2.15

$$S_{3\phi} = V_{AN} I_A^* + V_{BN} I_B^* + V_{CN} I_C^*$$

$$= (\sqrt{2}\angle 45^\circ)(1\angle{+10}) + (1\angle{-90})\left(\tfrac{1}{\sqrt{2}}\angle{+145}\right)$$

$$+ (1\angle 180)\left(\tfrac{1}{\sqrt{2}}\angle{-125^\circ}\right)$$

$$= \sqrt{2}\angle 55^\circ + \frac{1}{\sqrt{2}}\angle 55^\circ + \frac{1}{\sqrt{2}}\angle 55^\circ$$

$$= \left(\sqrt{2} + \frac{2}{\sqrt{2}}\right)\angle 55^\circ = (\sqrt{2}+\sqrt{2})\angle 55^\circ = \boxed{2\sqrt{2}\angle 55^\circ \text{ VA} \quad S_{3\phi}}$$

$$= 2.83\angle 55 \text{ VA}$$

$$= \underbrace{1.623}_{W} + j\underbrace{2.32}_{VAr}$$

Check I^2R, I^2X

Each Z, $R = \sqrt{2}\cos(55^\circ) = 0.811\ \Omega$

$jX = j\sqrt{2}\sin(55^\circ) = j1.159\ \Omega$

OK

$$P = |I_a|^2 R + |I_b|^2 R + |I_c|^2 R$$

$$= \left[(1)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2\right](0.811) = \left[1 + \tfrac{1}{2} + \tfrac{1}{2}\right](0.811) = 1.622\text{ W}$$

$$Q = \left[(1)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2\right](1.159) = (2)(1.159) = 2.32\text{ VAr}$$