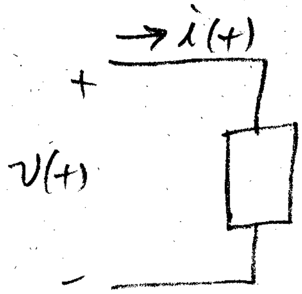
2.2. What do we know?

Single phase, $V_{RMS} = 100V$, $Max(p(t)) = 1707W$, $Min(p(t)) = -293W$.



$p(t) = v(t)\,i(t)$, $v(t) = V_p \cos(\omega t + \theta_V)$, $i(t) = I_p \cos(\omega t + \theta_I)$

$V_p = 100\sqrt{2}\ V$, $I_p = \sqrt{2}\, I_{RMS}$
(100 ↑ RMS)

Let's do b) first because usually we need P, Q before getting an equivalent circuit.

(b) P is the average value of $p(t)$.

$$p(t) = v(t)\,i(t) = \frac{V_p I_p}{2}\left[\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)\right]$$

So, $p(t)$ is a constant plus a double-frequency cosine. The average value of $p(t)$ is then simply the constant. Thus, $\boxed{P = \frac{V_p I_p}{2}\cos(\theta_V - \theta_I)}$ (1)

From the $p(t)$ equation, we can see that

$$Max(p) = \frac{V_p I_p}{2}\left[\cos(\theta_V - \theta_I) + 1\right]$$

$$Min(p) = \frac{V_p I_p}{2}\left[\cos(\theta_V - \theta_I) - 1\right]$$

Subtracting Min from Max yields

$$\boxed{Max(p) - Min(p) = \frac{V_p I_p}{2}\left[1 - (-1)\right] = V_p I_p} \quad (2)$$

Finally, since $p(t)$ is a sinusoidal function (with an offset), $\boxed{Avg(p(t)) = \frac{Max(p) + Min(p)}{2} = P}$ (3)

So, from ③, we get $\boxed{P = \frac{1707-293}{2} = 707W}$

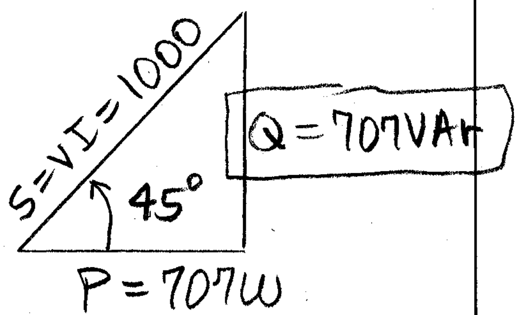
From ②, $V_P I_P = 1707-(-293) = 2000$, so we get

$I_P = \frac{2000}{100\sqrt{2}} = \frac{20}{\sqrt{2}}$, and so $\boxed{I = \frac{I_P}{\sqrt{2}} = 10A}$

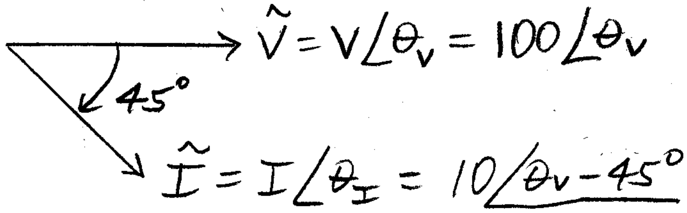
Now, from ①, $\underset{pf}{\cos(\theta_V-\theta_I)} = \dfrac{P}{\frac{V_P}{\sqrt{2}}\frac{I_P}{\sqrt{2}}} = \dfrac{P}{V_{RMS} I_{RMS}}$

$$\boxed{\cos(\theta_V-\theta_I) = \frac{707}{(100)(10)} = \underset{\cos(45^\circ)}{0.707}}$$

So, the power triangle is



Here, we assume the load is inductive (but the only real clue is in part (a) where it says RL model). The voltage and current phasors are



$\underbrace{S = VI^* = VI\angle\theta_V-\theta_I}_{\text{complex numbers}}$

A handy formula for Q comes from

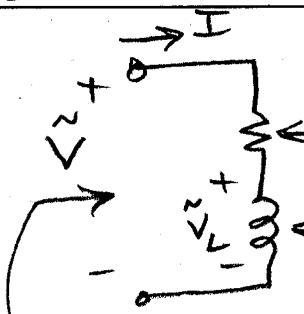
$$\underbrace{S^2 = P^2+Q^2}_{\text{magnitudes}}, \quad Q^2 = S^2-P^2 = \left(\frac{P}{pf}\right)^2 - P^2$$

$$Q^2 = P^2\left(\frac{1}{pf^2}-1\right)$$

$$\boxed{Q = \pm P\sqrt{\left(\frac{1}{pf}\right)^2-1}}$$

$Q = \pm 707\sqrt{\left(\frac{1}{0.707}\right)^2-1} = \pm 707$. Choose $+707$ assuming that the load is inductive.

(a)



$\leftarrow P = I^2 R,\ Q = 0$

$\leftarrow Q = I^2 X_L,\ P = 0$

Conservation of power – what flows in must be the (load P) + j (load Q)

$$S = V I^* = (100\angle\theta_v)(10\angle\theta_v - 45^\circ)^* = 1000\angle 45^\circ = P + jQ$$

$$P = 707\,W, \quad Q = 707\,VAr$$

$$P = I^2 R, \quad R = \frac{P}{I^2} = \frac{707}{10^2} = \boxed{7.07\,\Omega = R}$$

$$Q = I^2 X_L, \quad X_L = \frac{Q}{I^2} = \frac{707}{10^2} = \boxed{7.07\,\Omega = \omega L}$$

(ω not given, but it's probably $2\pi(60)$, so $L = 18.8\,mH$

(c) Examine the L. $\quad \tilde{V}_L = \tilde{V}\left[\frac{j\omega L}{R + j\omega L}\right]$

We know that $R + j\omega L = 7.07 + j7.07\,\Omega$. Another way to get $Z_{LOAD} = R + j\omega L$ is to use $Z_{LOAD} = \frac{\tilde{V}}{\tilde{I}}$

$$Z_{LOAD} = \frac{100\angle\theta_v}{10\angle\theta_v - 45^\circ} = 10\angle 45^\circ\,\Omega$$ checks

$$\text{So,}\quad \tilde{V}_L = \tilde{V}\left[\frac{j7.07}{10\angle 45}\right] = (100\angle\theta_v)(0.707\angle 45^\circ) = 70.7\angle\theta_v + 45^\circ$$

So, $v_L(t) = 70.7\sqrt{2}\cos(\omega t + \theta_v + 45^\circ)$

$\quad i_L(t) = 10\sqrt{2}\cos(\omega t + \theta_v - 45^\circ)$

$$P_L(t) = \frac{(70.7\sqrt{2})(10\sqrt{2})}{2}\left[\cancel{\cos}^{\,0}(90^\circ) + \cos(2\omega t + 2\theta_v)\right]$$

$$\text{MAX}(P_L) = (70.7)(10) = 707\,W$$ (which equals the Q!)