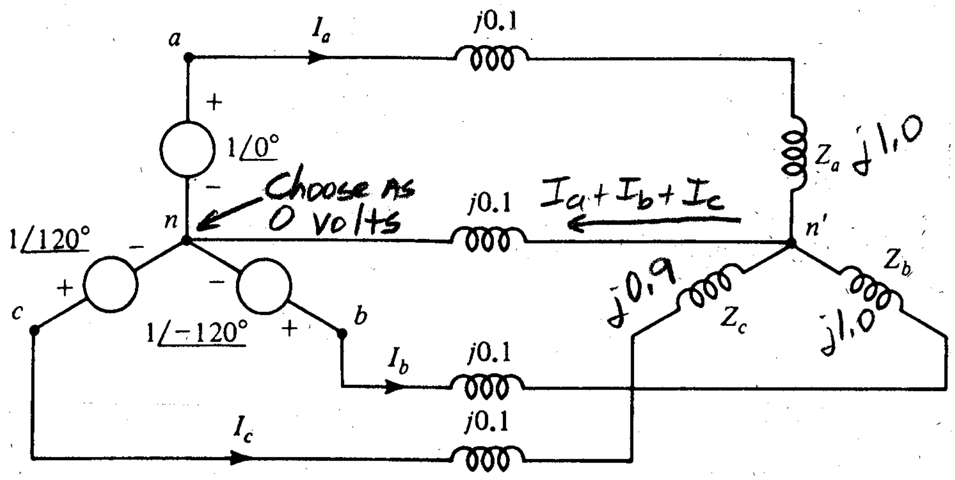
2.8



a) Unbalanced because $Z_a = Z_b = j\,1.0$, $Z_c = j0.9$ (but close to balanced). For unbalanced case, we have a conventional circuit problem – can write three mesh current equations, OR one node equation at N', choosing N as the reference. Obviously, one node equation is easiest!

$$\frac{V_{N'} - V_{aN}}{j0.1 + Z_a} + \frac{V_N' - \overset{0}{\cancel{V_N}}}{j0.1} + \frac{V_N' - V_{bN}}{j0.1 + Z_B} + \frac{V_N' - V_{CN}}{j0.1 + Z_c} = 0$$

Gather terms

$$V_N'\left[\frac{1}{j0.1 + j1.0} + \frac{1}{j0.1} + \frac{1}{j0.1 + j1.0} + \frac{1}{j0.1 + 0.9}\right] = \frac{V_{aN}}{j0.1 + j1.0} + \frac{V_{bN}}{j0.1 + j1.0} + \frac{V_{CN}}{j0.1 + j0.9}$$

$$V_N'\left[-j0.909 - j10 - j0.909 - j1\right] = \frac{1\angle 0}{j1.1} + \frac{1\angle -120}{j1.1} + \frac{1\angle 120}{j1.0}$$

$$V_N'\left[-j12.8\right] = -j0.909\left(1 - \tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}\right) - j1\left(-\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}\right)$$

$$-j12.8V_N' = -j0.455 - 0.787 + j0.5 + 0.866 = 0.079 + j0.045$$

$$V_N' = \frac{0.0909\angle 29.7^\circ}{12.8\angle -90^\circ} = 0.0071\angle 119.7^\circ$$

2.8, cont

Now, $\tilde{I}_a = \dfrac{\tilde{V}_{aN} - V_{N'N}}{j0.1 + j1.0} = \dfrac{1\angle 0 - 0.0071\angle 119.7}{j1.1}$

And so on. But $V_{N'N}$ is so close to zero that the usual balanced assumption is good.

$$\tilde{I}_a = \frac{\tilde{V}_{aN}}{j0.1 + j1.0} = \frac{1\angle 0}{j1.1} = -j0.909\angle 0 = 0.909\angle -90^\circ \text{ A}$$

$$\tilde{I}_b = \hat{I}_a \angle -120, \quad \hat{I}_c = \tilde{I}_a \angle 120^\circ$$

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