Problem 1. Determine the rms value of the stairstep time-domain voltage waveform shown below.

\[
V_{\text{rms}}^2 = \frac{1}{T} \left[ \frac{T}{4} a^2 + \frac{T}{4} (\frac{2a}{3})^2 + \frac{T}{4} \left( \frac{a}{3} \right)^2 + \frac{T}{4} (0)^2 \right]
\]
\[
= \frac{a^2}{4} + \frac{4a^2}{36} + \frac{a^2}{36} + 0
\]
\[
= a^2 \left( \frac{1}{4} + \frac{1}{9} + \frac{1}{36} \right)
\]
\[
= a^2 \left( \frac{9 + 4 + 1}{36} \right) = \frac{14}{36} a^2 = \frac{7}{18} a^2
\]

\[
V_{\text{rms}} = \sqrt{\frac{7}{18}} a = 0.624 a
\]
Problem 2. An automobile is inside the box shown below, and the box has two wires connected to the “outside world.” The voltage and current waveforms for the wires are

\[ v(t) = 100 \sin(\omega t + 10^\circ) + 20 \sin(3\omega t - 100^\circ) \text{ Volts} \]

\[ i(t) = 2 \sin(\omega t - 20^\circ) + 4 \sin(3\omega t + 40^\circ) + 3 \sin(5\omega t) \text{ Amperes} \]

Use the power expression for Fourier series to determine the average power \( P_{avg} \) that flows into the box from the outside world.

\[
P_{avg} = V_{avg} I_{avg} + \frac{1}{2} \sum_{k=1}^{\infty} V_k I_k \cos(S_k - \Theta_k)
\]

\[
= \frac{1}{2} \left[ 100 \cdot 2 \cos(10^\circ - (-20^\circ)) + 20 \cdot 4 \cos(-100^\circ - 40^\circ) \right]
\]

\[
= 100 \cos(30^\circ) + 40 \cos(-140^\circ)
\]

\[
= 86.6 - 30.6 = 56 \text{ W}
\]
Problem 3. The FFT of the ENS wall outlet voltage is shown. The horizontal “span” is 1kHz, and the “center frequency” is 500Hz. The vertical scale is 10dB per division.

Note – make sure that you show adequate marks and values on the figure to justify your answers.

a) Which two harmonics (above the fundamental) are the largest?

b) Estimate the magnitudes of each of those harmonics – express your answers in percent of the fundamental.

c) Use the results of part b) to estimate the total harmonic distortion (THD) of the ENS wall outlet voltage.

\[
\text{b) } 5^{\text{th}} \text{ harmonic is } 36 \text{ dB down from the } 1^{\text{st}} \\
7^{\text{th}} \text{ harmonic is } 40 \text{ dB down from the } 1^{\text{st}}
\]

\[
dB = 20 \log_{10} \frac{V_h}{V_1}, \quad \frac{dB}{20} = \log_{10} \frac{V_h}{V_1} \Rightarrow \frac{V_h}{V_1} = 10
\]

\[
\text{So, } \frac{V_5}{V_1} = 10^{-36/20} = 0.0158 \Rightarrow 1.58\%
\]

\[
\text{So, } \frac{V_7}{V_1} = 10^{-40/20} = 0.0100 \Rightarrow 1.00\%
\]

\[
\text{c) } \text{THD}_v \approx \sqrt{(1.58)^2 + (1.00)^2} = 1.87\%
\]
Problem 1. A DBR is plugged into a 60Hz wall output and is operating in steady state. The AC current has triangular pulses, is half-wave symmetric, varies between ±5A, and has zero average value. The average voltage on the capacitor is 160V. C is very large and has very low ripple voltage. The load power is constant. Determine the average power delivered to the load. Explain your reasoning. Hint: use the DC side.

\[ i_{dc} \text{ looks like } i_{ac}, \text{ except that the negative \ pulses \ are \ positive.} \]

The average value of \( i_{dc} \) is

\[ \frac{1}{8.333 \text{ msec}} \left[ 2.5 \left( \frac{2 \text{ msec}}{8.333 \text{ msec}} \right) + 0 \left( \frac{6.333 \text{ msec}}{8.333 \text{ msec}} \right) \right] \]

\[ = 2.5 \left( \frac{2}{8.333} \right) = 0.16 \text{ A} \]

From \( P = V_{avg} I_{avg} + \frac{1}{2} \sum_{K=1}^{\infty} V_K I_K \cos(S_K - \Theta_K) \),

\[ = (160 \text{ V})(0.16 \text{ A}) \]

\[ P = 96 \text{ W} \]

**Version b, 134.4W**

**c, 194W**

**d, 202W**
Problem 2. Measured solar radiation data, sun zenith angles, and incidence angles for our fixed panels for Feb. 16 are shown in the table.

A. Estimate the incident power (per square meter of surface area) on our panels at 10:30. Use

\[
P_{\text{incident}} = \left[ \frac{58}{DH} \cdot \frac{544 - 58}{(GH - DH)} \cdot \cos\left(\frac{\theta_{\text{zenith}}}{\cos\left(\frac{\theta_{\text{sun}}}{0.588}\right)}\right) \right] \text{W/m}^2 = 691 \text{ W/m}^2 \tag{A}
\]

B. Repeat part A, but assume that the panels are unbolted and then directed toward the sun so that the incident power is maximized.

Using 45° tilt, 190° azimuth

\[
\theta = \frac{58 + \frac{544 - 58}{0.588}}{1} = 885 \text{ W/m}^2 \tag{B}
\]

<table>
<thead>
<tr>
<th>Feb. 16</th>
<th>Global Horizontal W/m²</th>
<th>Diffuse Horizontal W/m²</th>
<th>Zenith Angle degrees</th>
<th>Incidence Angle degrees</th>
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</thead>
<tbody>
<tr>
<td>1000</td>
<td>466</td>
<td>57</td>
<td>59</td>
<td>47</td>
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<td>544</td>
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<tr>
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<tr>
<td>1600</td>
<td>455</td>
<td>55</td>
<td>64</td>
<td>40</td>
</tr>
</tbody>
</table>

Version b, 11:30
\[
64 + \frac{686-64}{0.682} \cdot 0.906 = 890 \text{ W/m}^2
\]
\[
64 + \frac{686-64}{0.682} = 976 \text{ W/m}^2
\]

Version c, 14:30
\[
60 + \frac{675-60}{0.643} \cdot 0.951 = 970 \text{ W/m}^2
\]

\[
60 + \frac{675-60}{0.643} = 1016 \text{ W/m}^2
\]

Version d, 15:30
\[
57 + \frac{590-57}{0.839} = 894 \text{ W/m}^2
\]

\[
57 + \frac{590-57}{0.839} = 995 \text{ W/m}^2
\]

1 or 2 points off each A & B, depending on severity of the mistake.
Problem 3. The I-V curve for one of our solar panel pairs at one particular sun/sky/orientation condition is given below.

A. If a 20Ω load resistor is connected directly to the panel pair, how much power will the resistor absorb? Hint: use a load line for the resistor.

\[
\text{Power} = \frac{V^2}{R} = \frac{(36.5)^2}{20} = 67 \text{ W}
\]

B. If a boost converter is connected between the panel pair and the load resistor, what duty cycle D will cause the panel pair voltage to drop to 24V? Hint: the equivalent resistance at the input of the boost converter is

\[R_{\text{equiv}} = R_{\text{load}} \cdot (1-D)^2\]

\[R_{\text{equiv}} = \frac{24V}{5A} = 4.8 \text{ Ω}\]

\[(1-D)^2 = \frac{R_{\text{equiv}}}{R_{\text{load}}} = \frac{4.8}{20} = 0.24\]

\[1-D = \sqrt{0.24} = 0.490\]

\[D = 1 - 0.490 = 0.51\]
Problem 3. The I-V curve for one of our solar panel pairs at one particular sun/sky/orientation condition is given below.

A. If a 40Ω load resistor is connected directly to the panel pair, how much power will the resistor absorb? Hint: use a load line for the resistor. 
\[ P = \left( \frac{37.8}{40} \right)^2 = 36 \text{W} \]

B. If a boost converter is connected between the panel pair and the load resistor, what duty cycle D will cause the panel pair voltage to drop to 24V? Hint: the equivalent resistance at the input of the boost converter is \( R_{\text{equiv}} = R_{\text{load}} \cdot (1-D)^2 \).

\[(1-D)^2 = \frac{R_{\text{equiv}}}{R_{\text{LOAD}}} = \frac{4.8}{40} = 0.12 \]

\[(1-D) = 0.346, \quad D \approx 0.654 \]
Problem 3. The I-V curve for one of our solar panel pairs at one particular sun/sky/orientation condition is given below.

A. If a 10Ω load resistor is connected directly to the panel pair, how much power will the resistor absorb? Hint: use a load line for the resistor.

\[ P = \frac{(33.5)^2}{10} = 112 \text{W} \]

B. If a boost converter is connected between the panel pair and the load resistor, what duty cycle \( D \) will cause the panel pair voltage to drop to 24V? Hint: the equivalent resistance at the input of the boost converter is \( R_{\text{equiv}} = R_{\text{load}} \cdot (1 - D)^2 \).

\[
\frac{(1-D)^2}{10} = \frac{4.8}{10} = 0.48
\]

\[
(1-D) = 0.693
\]

\[
D = 0.307
\]
Problem 3. The I-V curve for one of our solar panel pairs at one particular sun/sky/orientation condition is given below.

A. If an 8Ω load resistor is connected directly to the panel pair, how much power will the resistor absorb? Hint: use a load line for the resistor.

\[ P = (31.5V)(3.9A) = 123W \]

B. If a boost converter is connected between the panel pair and the load resistor, what duty cycle \( D \) will cause the panel pair voltage to drop to 24V? Hint: the equivalent resistance at the input of the boost converter is \( R_{\text{equiv}} = R_{\text{load}} \cdot (1 - D)^2 \).

\[
4.8 = 8(1-D)^2 \\
(1-D)^2 = \frac{4.8}{8} = 0.60 \\
1-D = 0.775 \\
D = 0.225
\]
Problem 4. Much can be learned from the inductor current of a DC-DC converter. Consider the current waveform for a boost converter inductor \( L \), as shown below.

If the converter input voltage is 50V, and the operating frequency is 100kHz, what is the inductance of \( L \)?

\[
L = \frac{V_{IN}}{\frac{d}{dt}i_{IN}} = \frac{50 \text{ V}}{\frac{4A}{3 \mu s}} = \frac{150}{4} \mu \text{H} = 37.5 \mu \text{H}
\]

\[\text{Version b,} \quad \frac{d}{dt}i_{IN} = \frac{4A}{2 \mu \text{s}} \quad L = \frac{50}{2} \mu \text{H} = 25 \mu \text{H}\]

\[\text{Version c,} \quad \frac{d}{dt}i_{IN} = \frac{4A}{6 \mu \text{s}} \quad L = \frac{50}{4} \mu \text{H} = \frac{250}{4} \mu \text{H} = 75 \mu \text{H}\]

\[\text{Version d,} \quad \frac{d}{dt}i_{IN} = \frac{4A}{7 \mu \text{s}} \quad L = \frac{50}{4} \mu \text{H} = \frac{350}{4} \mu \text{H} = 87.5 \mu \text{H}\]
Problem 1. The output filter of your inverter is shown below. Assume that the voltage on the inverter side has a 10V rms, 40kHz component. How many rms volts of 40kHz appear across the 10μF output capacitor? Note – no load is attached.

\[ V_{out} = \frac{Z_c}{Z_L + Z_c} = \frac{j\omega C}{j\omega L + \frac{1}{j\omega C}} = \frac{1}{j\omega C} \left( \frac{1}{j(\omega L - \frac{1}{\omega C})} \right) \]

\[ \frac{V_{out}}{V_{in}} = \frac{-1}{\omega C} = \frac{-1}{w^2LC - 1} = \frac{1}{1 - w^2LC} \]

\[ \frac{V_{out}}{V_{in}} = \frac{1}{1 - (2\pi \times 40000)^2 (100 \times 10^{-6}) (10 \times 10^{-6})} = \frac{1}{1 - 63.2} = -0.01609 \]

\[ V_{out} = V_{in} \times (-0.01609) = \boxed{0.1609 \text{ } V_{rms}} \]

Other versions:

- 30kHz → 0.290 Vrms
- 50kHz → 0.1024 Vrms
Problem 2. The figure shown below is taken from the lab document. The approx. 2μs time between the two vertical bars is the delay time between the moment that the 12V driver output “goes high” and the actual “turn on” of the MOSFET (which occurs when V_{GS} ≈ 4V). The MOSFET gate capacitance C is a few nC, and the series gate firing resistor R is 1200Ω.

Suppose you want to double the 2μs delay time to add more blanking. Estimate the new series gate firing resistance needed to do this. Hint – in the early portion of the V_{GS} rising curve, its slope is

\[ \frac{V_{\text{final}}}{\tau} = \frac{12}{RC} \]

Volts per second. Give good reasons for your answer!

\[ \frac{dV}{dt} = \frac{12}{RC} \quad \text{At the early part of the rising curve} \]

\[ \frac{\Delta V}{\Delta t} = \frac{12}{RC} \quad , \quad \Delta t = \frac{(\Delta V)RC}{12} \]

For a fixed \( \Delta V = 4V \), then \( \Delta t \propto R \)

So, to double \( \Delta t \), double \( R \), 1200\( \cdot \)2 = 2400Ω

Other versions

To triple, 1200\( \cdot \frac{3}{2} = 3600\Omega \), To lower from 2μs to 1μs, 1200\( \cdot \frac{1}{2} = 600\Omega \)

Suppose that you daisy chain five optoisolators (instead of two) in series, with each one operating at the circled point shown on the curve (1.6V LED voltage, 15mA LED current). What value of series resistance (in ohms) is needed to do this?

\[
\frac{24-8.0}{R} = 0.015 \text{ A}
\]

\[
R = \frac{24-8.0}{0.015} = \frac{16}{0.015} = 1067\,\Omega
\]

Other versions, four optos,

\[
R = \frac{24-(1.6)(4)}{0.015} = \frac{24-6.4}{0.015} = 1173\,\Omega
\]

Six optos,

\[
R = \frac{24-(1.6)(6)}{0.015} = \frac{24-9.6}{0.015} = 960\,\Omega
\]
Problem 4. Show me how familiar you are with H-bridge connections. Use your pencil to neatly “wire up” only the requested portions of the H-bridge. Leave out the fuses. The requested portions are

- Draw in the input filter capacitor.
- Draw in the output filter inductor and capacitor.
- Draw in all the 16-gauge (or 14-gauge) wires that are attached to the A⁺ and A⁻ terminal blocks. Show the terminations of both ends of these wires. When wires cross, make it clear if connections are made at the crossings or not. Do not use colors.

If you need to re-draw, use the duplicate figure at the bottom of the page, and then cross out the top figure.

Any H-Bridge connections to a gate \( \Rightarrow \frac{1}{2} \) credit
10 multiple choice questions. Four 8.5x11” note sheets permitted (both sides). No laptop computers. Show all work on these pages. You must show sufficient reasoning, equations, and steps to justify your answer. Circle the correct answer. If you believe that the correct answer is not among them, then choose “Other” and write in your answer. No partial credit. Do not un staple.

Problem 1. A variable resistor is connected to the output of an ideal DC-DC boost converter. \( V_{in} = 40V, V_{out} = 100V, f = 50kHz, L = 100\mu H. \) \( V_{in} \) and \( V_{out} \) are held constant. The variable resistor ohm value is initially low enough that the converter is operating in continuous conduction.

Slowly the load resistor ohms are increased until the converter reaches the continuous/discontinuous boundary. At what resistance does this occur? \( (1\Omega \text{ accuracy}) \)

\[
\frac{V_{out}}{V_{in}} = \frac{1}{1-D}, \quad V_{in} = V_{out}(1-D)
\]

\[
(1-D) = \frac{V_{in}}{V_{out}} = \frac{40}{100} = 0.4, \quad D = 0.6
\]

<table>
<thead>
<tr>
<th>104Ω</th>
<th>78Ω</th>
<th>52Ω</th>
<th>156Ω</th>
<th>Other</th>
</tr>
</thead>
</table>

At the boundary of continuous/discontinuous, current \( I_{IN} \) looks like

\[
ZI_{IN} = \left(\frac{V_{IN}}{L}\right)DT
\]

\[
I_{IN} = \frac{V_{IN}DT}{2L} = 2.4A
\]

So \( I_{OUT} = (1-D)I_{IN} = 0.4I_{IN} = 0.96A \)

\[
R = \frac{V_{OUT}}{I_{OUT}} = \frac{100}{0.96} = 104.2\Omega
\]

Check. \( P_{IN} = V_{IN}I_{IN} = (40)(2.4) = 96W \)

\( P_{OUT} = V_{OUT}I_{OUT} = (100)(0.96) = 96W \)

\[
P_{OUT} = \frac{V_{OUT}^2}{R} = \frac{(100)^2}{104.2} = 96W
\]
Problem 2. The periodic voltage waveform shown is applied across a 10Ω resistor. The period of the waveform is T seconds. For the first period, \( v(t) = t^2 \), and the waveform keeps repeating. If the average power to the resistor is 100W, determine T (in seconds). (0.1 second accuracy)

\[
V_{\text{rms}}^2 = \frac{1}{T} \int_0^T v^2(t) \, dt
\]

We have \( v = t^2 \), so \( v^2 = t^4 \)

<table>
<thead>
<tr>
<th>8.4 sec.</th>
<th>54.8 sec.</th>
<th>70.7 sec.</th>
<th>10.0 sec.</th>
<th>Other</th>
</tr>
</thead>
</table>

\[
V_{\text{rms}}^2 = \frac{1}{T} \int_0^T t^4 \, dt = \frac{\frac{t^5}{5} \bigg|_0^T}{5} = \frac{T^5}{5T} = \frac{T^4}{5}
\]

\[
P = \frac{V_{\text{rms}}^2}{R} = \frac{T^4}{5R} \quad T^4 = 5PR, \quad T^2 = \sqrt{5PR}
\]

\[
T = 8.11 \text{ sec}
\]
Problem 3. The series circuit shown has been operating in steady-state for a long time. $v(t) = 12(\sin 1000t + 2)$ volts, $R = 10\Omega$, $L = 10\text{mH}$, $C = 100\mu\text{F}$. A DC voltmeter is connected across the capacitor with the polarity shown. What voltage will the meter read? (0.1 volt accuracy)

\[ + \quad v(t) \quad \text{A superposition problem with 1000 rad/sec circuit, and DC circuit} \quad - \]

\begin{tabular}{|c|c|c|c|c|}
\hline
-15.0 volts & 24.0 volts & 12.0 volts & 15.0 volts & Other \hline
\end{tabular}

The average values come from the "DC" equivalent circuit. The $C$ is "open" so no current flows.

\[ R = 10 \quad L \text{ is a short ckt} \]

\[ 24\text{V} \quad \text{OA} \quad 24\text{V} \quad \text{C is an open ckt} \]

The entire 24V appears across C

Also, using properties of L's & C's in S.S.

- The average voltage across an L is zero
- The average current thru a C is zero

\[ V_{\text{avg}} = \frac{24}{2} \quad \text{AVERAGE VALUES} \]

\[ I = 0 \quad -24\text{V} \]

\[ I = 0 \quad \text{I = 0} \]

\[ + \quad 0^- \quad + \quad 0^- \quad \text{MM} \quad \text{MM} \]

\[ + \quad - \quad 0^- \quad \text{MM} \quad \text{MM} \]
Problem 4. A bridge rectifier module is used to charge a battery. Starting from zero, \( V_p \) is very slowly increased with a variac until the "fast-blow" fuse blows. At what \( V_p \) does this occur? (assume that the fuse blows instantaneously when 5A is reached) \((0.1 \text{ volt accuracy})\)

\[
\begin{align*}
&0.2\Omega \\
&0.3\Omega \\
&5A \text{ fuse} \\
&V_p \sin(120\pi t) \text{ volts} \\
&+ \\
&- \\
&+ \quad 13.6 \text{ volt battery} \\
&- \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>15.1 volts</th>
<th>14.6 volts</th>
<th>16.1 volts</th>
<th>18.6 volts</th>
<th>Other</th>
</tr>
</thead>
</table>

During conduction, the circuit is

\[
\begin{align*}
&0.3\Omega \quad 5A \quad 0.2\Omega \\
&\downarrow \\
&V_p \sin(\omega t) \quad \bigcirc \\
&i \\
&13.6 V \\
&- \\
\end{align*}
\]

The peak current is \( i_{\text{peak}} = \frac{V_p - 13.6}{0.3 + 0.2} \)

So, with \( i_{\text{peak}} = 5A \), we get

\[
V_p - 13.6 = (5)(0.3 + 0.2) = 2.5
\]

\[
V_p = 13.6 + 2.5 = 16.1 \text{ V}
\]
Problem 5. A 16,000μF capacitor is charged to 40V. At t = 0, the switch closes, connecting the capacitor to a 500Ω resistor. How long does it take for the capacitor to give up 2/3 of its stored energy? (0.1 second accuracy)

\[ V(t) = V_{\text{INIT}} e^{-t/\tau}, \quad t > 0 \]

\[ V_{\text{INIT}} = 40V \]

| 8.8 sec. | 6.9 sec. | 1.6 sec. | 4.4 sec. | Other |

Let \( V_{1/3} \) be the voltage where \( C \) still has \( \frac{1}{3} \) of its initial energy.

\[ \frac{1}{2} C V_{1/3}^2 = \frac{1}{3} C V_{\text{INIT}}^2 \]

\[ V_{1/3}^2 = \frac{V_{\text{INIT}}^2}{3}, \quad V_{1/3} = \frac{V_{\text{INIT}}}{\sqrt{3}} \]

Solve for \( t = t_{1/3} \)

\[ V_{1/3} = V_{\text{INIT}} e^{-t_{1/3}/\tau}, \quad \frac{V_{\text{INIT}}}{\sqrt{3}} = V_{\text{INIT}} e^{-t_{1/3}/\tau} \]

\[ e^{-t_{1/3}/\tau} = \frac{1}{\sqrt{3}}, \quad -\frac{t_{1/3}}{\tau} = \ln\left(\frac{1}{\sqrt{3}}\right) = -0.5493 \]

\[ t_{1/3} = 0.5493 \tau = 0.5493(500)(16000 \times 10^{-6}) \]

\[ t_{1/3} = 4.39 \text{ sec} \]

Check \( 40 e^{-4.39/500.16000.10^{-6}} = 23.1V, \quad \frac{23.1}{40} = 0.578, \quad \frac{1}{0.578} = 1.73 \checkmark \)
Problem 6. Consider the inverter that you built. Assume that you are powering its MOSFET driver chips with 18Vdc instead of 12Vdc. MOSFET turn-on occurs when $V_{GS} = 4V$, and you need a turn-on delay of 2μsec to achieve adequate blanking. The capacitance of the MOSFET gate is $5(10^{-9}) \text{F}$. What value of turn-on series resistance should you use in the output of your driver chips? (three digit accuracy)

$$V(t) = 18(1 - e^{-t/\tau})$$

$$4 = 18 \left( 1 - e^{-\frac{2 \mu s}{\tau}} \right)$$

$$\frac{4}{18} = 1 - e^{-\frac{2 \mu s}{\tau}}$$

$$e^{-\frac{2 \mu s}{\tau}} = 1 - \frac{4}{18} = \frac{18 - 4}{18} = \frac{14}{18} = \frac{7}{9}$$

$$-\frac{2 \mu s}{\tau} = \ln \left( \frac{7}{9} \right)$$

$$\tau = \frac{-2 \mu s}{\ln \left( \frac{7}{9} \right)} = 1.96 \mu s$$

$$R = \frac{\tau}{C} = \frac{7.96 \mu s}{0.005 \mu F} = 1592.5 \Omega$$

Check: $18 \left( 1 - e^{-\frac{2 \times 10^{-6}}{(1592)(5 \times 10^{-9})}} \right) = 4 \checkmark$
Problem 7. The time-domain waveform and FFT of the wall outlet current drawn by a PC are shown. The vertical scale is 10dB per division. If the fundamental current (i.e., 60Hz current) is 1 Arms, what is the rms magnitude of the 11th harmonic? (0.01A accuracy)

\[ 11^{\text{th}} \text{ harmonic} = 11 \cdot (60) = 660 \text{ Hz} \]

\[ -30 = 20 \log_{10} \frac{I_{11}}{I_1}, \quad \frac{-30}{20} = \log_{10} \frac{I_{11}}{I_1} \]

\[ -3/2 = \frac{I_{11}}{I_1}, \quad I_{11} = I_1 \cdot 10^{-3/2} = 0.0316 \cdot 1 \]

\[ 1 \text{A RMS} \]

So \[ I_{11} \approx 0.03 \text{A RMS} \]
Problem 8. Assume that the inverter you built is operated as follows:

- The DC input voltage to the H-Bridge is 20Vdc
- The peak value of the 25kHz triangle wave generator chip is 4V
- You connect a pure 500Hz sinusoidal voltage to the Vcont jack. The peak value is adjustable.
- Your 500Ω Vcont potentiometer is set to the maximum
- Your inverter output drives a single high-power 8Ω speaker
- The output of your inverter is a low-distortion 500Hz sinewave
- The output filter has no affect on the 500Hz signal

What peak value of Vcont will result in 10W average power to the speaker? (0.1 volt accuracy)

<table>
<thead>
<tr>
<th>8.9 volts</th>
<th>2.5 volts</th>
<th>1.8 volts</th>
<th>0.6 volts</th>
<th>Other</th>
</tr>
</thead>
</table>

\[ M_a = \frac{V_{\text{cont peak}}}{V_{\text{tri peak}}} \]

\[ P_{\text{INV}} = \frac{V_{\text{rms INV}}^2}{R_{\text{speaker}}} \]

\[ V_{\text{rms INV}} = \frac{V_{\text{dc}}}{\sqrt{2}} \cdot M_a \]

\[ P_{\text{INV}} = \left( \frac{V_{\text{dc}}}{\sqrt{2}} \cdot M_a \right)^2 = \left( \frac{V_{\text{dc}}}{\sqrt{2}} \cdot \frac{V_{\text{cont peak}}}{V_{\text{tri peak}}} \right)^2 \]

\[ \frac{V_{\text{dc}}}{\sqrt{2}} \cdot \frac{V_{\text{cont peak}}}{V_{\text{tri peak}}} = \sqrt{P_{\text{INV}} R_{\text{speaker}}} \]

\[ V_{\text{cont peak}} = \frac{V_{\text{tri peak}} \sqrt{2 P_{\text{INV}} R_{\text{speaker}}}}{V_{\text{dc}}} \]

\[ V_{\text{cont peak}} = \frac{(4) \sqrt{2} (10) (8)}{20} = 2.53 \text{V} \]

Check. \( M_a = \frac{2.53}{4} = 0.633 \), \( V_{\text{rms}} = \frac{20}{\sqrt{2}} (0.633) = 8.95 \text{V} \)

\[ \frac{V_{\text{rms}}^2}{R} = \frac{(8.95)^2}{8} = 10 \text{W} \quad \checkmark \]
Problem 9. An ideal DC-DC buck converter interfaces one of our solar panel pairs to a car battery. The Thevenin equivalent circuit of the battery is shown. Your objective is to charge the battery while holding the solar panels at their maximum power point. At what duty cycle $D$ must the buck converter operate to achieve this? (0.01 accuracy)

Check:

\[ V_p D = 28 \times 0.546 = 15.3 \]
\[ \frac{I_p}{D} = 4.6 \frac{0.546}{0.546} = 8.42 \]
\[ 15.3 = (8.42)(0.2) + 13.6 \]

\[ 0K \]

Max power point is 28V, 4.6A

| $D = 0.49$ | $D = 0.43$ | $D = 0.55$ | $D = 0.48$ | Other ______________ |

From the battery side, we have

\[ KVL, \quad V_p D - 0.2 \frac{I_p}{D} - 13.6 = 0 \]

\[ *D, \quad V_p D^2 - 0.2 I_p - 13.6 D = 0 \]

\[ \frac{d}{dV_p} \quad D^2 - \frac{0.2 I_p}{V_p} - \frac{13.6 D}{V_p} = 0 \]

\[ D^2 - \frac{13.6}{V_p} D - \frac{0.2 I_p}{V_p} = 0 \]

\[ D = \frac{0.486 \pm \sqrt{(0.486)^2 - 4(1)(-0.0324)}}{2} \]

\[ D = 0.546, \text{ feasible} \]
Problem 10. The incident power curve for our panels on April 25, 2007 is shown below. Use the curve, and assume a panel efficiency of 14%, to estimate the kWh of electric energy per square meter that one of our solar panel pairs could have produced on that day if it was max-power loaded. (0.1 kWh/m² accuracy)

![Incident Power on Panel]

- Each block is 100 WH incident

| 7.2 kWh/m² | 0.9 kWh/m² | 7.1 kWh/m² | **1.0 kWh/m²** | Other |

\[ \text{Triangle A, } \frac{1}{2}bh = \frac{1}{2}b(950) = 2850 \text{ Wh} \]
\[ \text{Rectangle B, } bh = (2.25)(950) = 2138 \text{ Wh} \]
\[ \text{Triangle C, } \frac{1}{2}bh = \frac{1}{2}(4.75)(950) = 2256 \text{ Wh} \]
\[ \frac{7444 \text{ Wh}}{0.14 \text{ efficiency}} \]
\[ 1014 \text{ Wh} \]
\[ \approx 1.0 \text{ kWh} \]