6.2

$V_a I_a^* = S_g, \quad I_a^* = \dfrac{S_g}{V_a} = \dfrac{1\angle 45}{1\angle 0} = 1\angle 45, \quad I_a = 1\angle -45$

Find $a' = V_a + r_a I_a + jX_q I_a$ to locate the q-axis

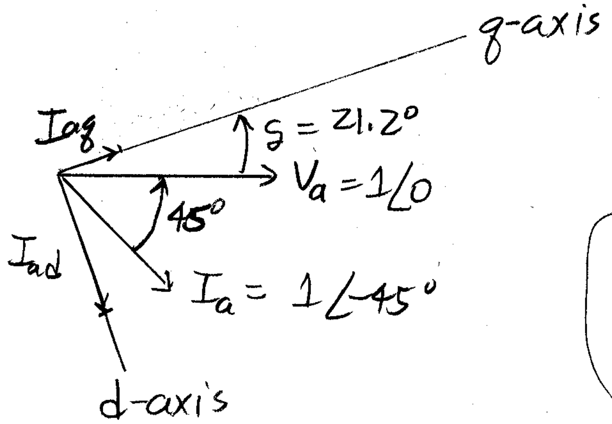
$= 1\angle 0 + (0) I_a + j0.9(1\angle -45) = 1\angle 0 + 0.9\angle 45$

$= (1 + j0) + (0.636 + j0.636) = 1.636 + j0.636$

$\boxed{a' = 1.755\angle 21.2^\circ \text{ V.}}$

So, since V_a has Ref. angle 0°, $\boxed{\delta = 21.2^\circ}$

Find I_{ad}, I_{aq}. $I_{ad} = |I_a| \sin(\underbrace{\delta - \angle I_a}_{(21.2+45)}) = \boxed{0.915 \text{ pu} = I_{ad}}$



$I_{aq} = |I_a| \cos(\delta - \angle I_a)$
$= 0.404 \text{ pu}$

So, $I_{ad} = 0.915\angle \cancel{\delta - 90^\circ}$
$= 0.915\angle -68.8^\circ$
$I_{aq} = 0.404\angle \delta = 21.2^\circ$

Find $E_a = V_a + rI_a + jX_d I_{ad} + jX_q I_{aq}$

$= 1 + j1.6(0.915\angle 68.8) + j0.9(0.404\angle 21.2)$

$= 1 + 1.464\angle 21.2 + 0.364\angle 111.2$

$= 1 + 1.365 + j0.529 - 0.1316 + j0.339$

$= 2.23 + j0.868$

$\boxed{E_a = 2.39\angle 21.2 \text{ pu}}$ (Angle checks with δ)

overexcited
because $\angle I_a$ is lagging, so E_a is large

6.4

$$S_g = V_a I_a^* = (1\angle 0)(1\angle{+60})^* = 1\angle{-60}$$

$$a' = V_a + r_a I_a + jX_q I_a = 1 + (0.1)(1\angle 60) + j(0.6)(1\angle 60)$$

$$a' = 1 + 0.1\angle 60 + 0.6\angle 150 = 1 + 0.05 + j0.0866 - 0.520 + j0.3$$

$$a' = 0.53 + j0.387 = 0.656\angle 36.1^\circ$$

So, the q-axis is at 36.1°

$$|I_{aq}| = |I_a|\cos(\delta - \angle I_a) = 1\cos(36.1 - 60) = 1\cos(-23.9)$$

$$\boxed{I_{aq} = 0.914\angle\delta = 0.914\angle 36.1}$$

$$|I_{ad}| = |I_a|\sin(\delta - \angle I_a) = 1\sin(36.1 - 60) = -0.405$$

$$I_{ad} = -0.405\angle 36.1 - 90^\circ = -0.405\angle{-53.9} = 0.405\angle 126.1$$

$$I_{ad} = 0.405\angle 126.1$$

$$\tilde{E}_a = V_a + r_a I_a + jX_d I_{ad} + jX_q I_{aq}$$

$$= 1 + (0.1)(1\angle 60) + j(1.0)(0.405\angle 126.1) + j(0.6)(0.914\angle 36.1)$$

$$= 1 + 0.1\angle 60 + 0.405\angle 216.1 + 0.548\angle 126.1$$

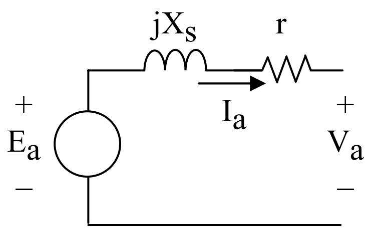
$$= 1 + 0.05 + j0.0866 - 0.327 - j0.239 - 0.323 + j0.443$$

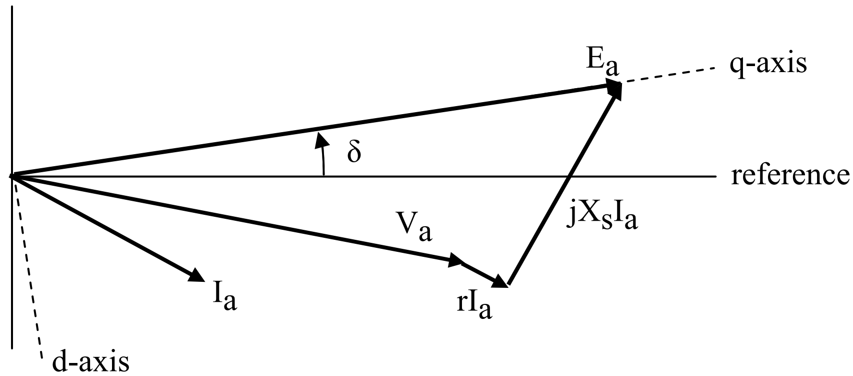
$$= 0.400 + j0.291$$

$$= 0.495\underbrace{\angle 36.1}_{\text{checks with } \delta}$$ ← Underexcited because $\angle I_a$ is leading so E_a is small

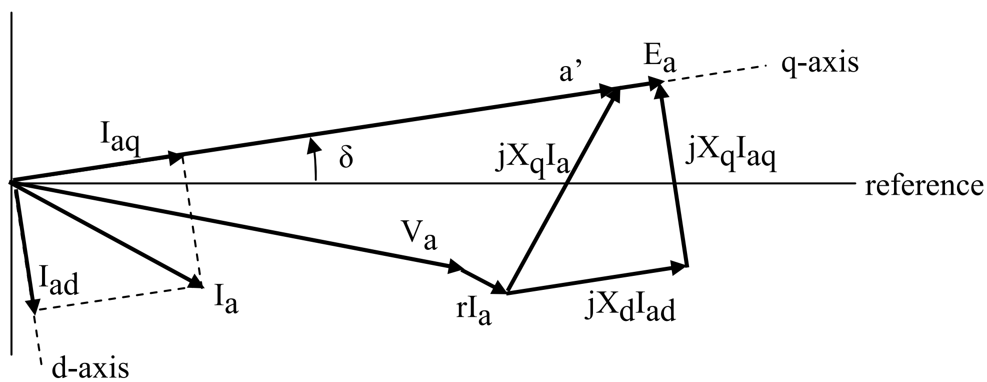
Steady State Model for Round Rotor Machine

V_a and I_a are the terminal voltage and current. X_s is the synchronous reactance. Resistance r is the stator resistance. E_a is the Thevenin equivalent voltage behind synchronous reactance.





Steady-State Model for Salient Pole Machine
(No equivalent circuit. Phasor diagram only)



Steps when V_a and I_a are known:

- With V_a, I_a, r, and X_q, find phasor a' to locate the q-axis.
- Then, compute I_a projections I_{aq} and I_{ad} where

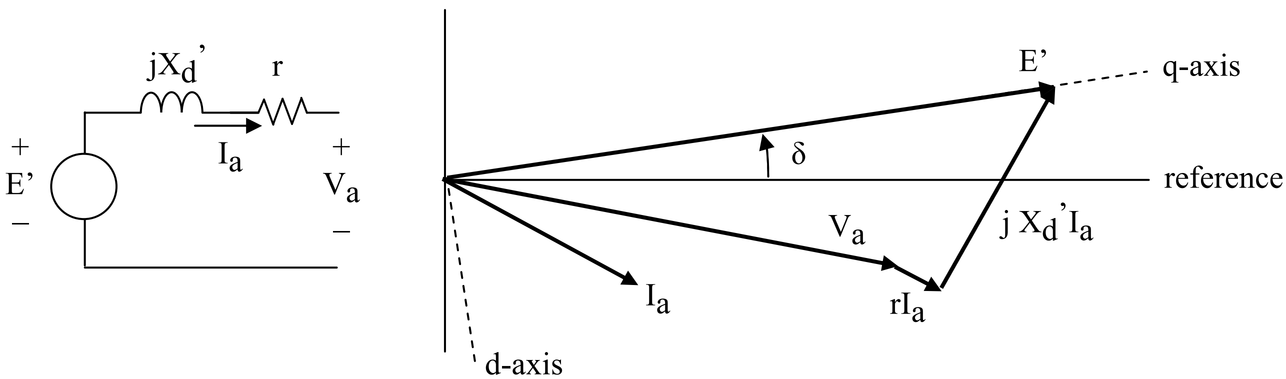
  I_{aq} has magnitude $|I_a|\cos(\delta - \angle I_a)$ and has phase angle δ,

  I_{ad} has magnitude $|I_a|\sin(\delta - \angle I_a)$ and has phase angle $(\delta - 90°)$,

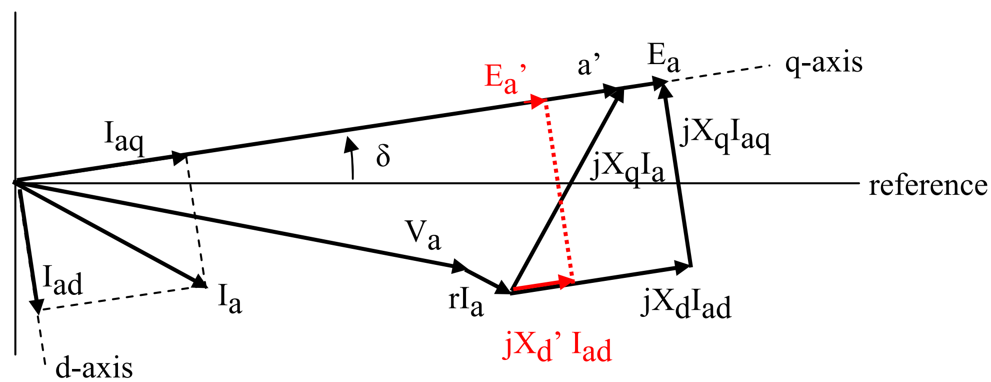
  and where $\angle I_a$ is the angle of I_a.
- Then, find E_a.

Transient Stability Machine Model 1
Constant Voltage Magnitude E' Behind Transient Reactance X_d'

This model is like the round rotor synchronous model, except that the transient reactance is used instead of the synchronous reactance.



Transient Stability Machine Model 2
Salient Pole Rotor



Steps when V_a and I_a are known: Begin with same steps used in the salient pole synchronous machine model. Then, use X_d' and I_{ad} to find E_a'.

The magnitude of E_a' varies according to $\frac{d\left|E_a'\right|}{dt} = \frac{1}{T_{do}'}\left(E_{fd} - \left|E_a\right|\right)$, where T_{do}' is the direct-axis transient open circuit time constant, and E_{fd} is the field voltage (as seen from the stator). As a first approximation, E_{fd} is treated as a constant whose initial value is the same as the initial value of $\left|E_a\right|$.