Problem 1. Capacitor inrush and the voltage ringing it causes are responsible for many power quality problems. Consider one phase of a 13.8kV (line-to-line rms) distribution system. A 600kVAR (total three phase) grounded-wye capacitor bank is about to be switched “on.” Each phase of the capacitor bank has a residual voltage of 10kV (line-to-neutral actual volts).

a. Determine the capacitor C in μF (each phase).

b. Assume a lossless power system. If the available short circuit current at the capacitor bus is 4000 Arms, determine the L of the system in mH (each phase).

c. Assume a lossless power system. Assume that the electrical closing occurs when the voltage across the switch is greatest. Write the time-domain expressions for capacitor current and voltage when the switch closes. As your last step, put numerical values in your equations.

d. If the normalized damping ratio zeta, \( \zeta = \frac{\alpha}{\omega_0} \), is observed from the recorded overshoot to be 0.25, what is the actual resistance of the power system?

\[
(2) \quad \alpha = \omega L = \frac{V_{LN}^2}{R_{rms}} \rightarrow C_{14} = \frac{\alpha}{\omega} \frac{V_{LN}^2}{R_{rms}} = \frac{600 \times 10^{3/1}}{(120\pi)(13800/\sqrt{3})^2} = 8.36 \mu F
\]

\[
(2) \quad I_{sc} = \frac{V_{LN}}{R_{rms} X_{sys}} \quad X_{sys} = \omega L = \frac{V_{LN}}{R_{rms}} \quad \frac{I_{sc}}{R_{rms}} = \frac{(13800/\sqrt{3})}{4000} = 1.99 \Omega
\]

\[
L_{sys} = \frac{X_{sys}}{120\pi} = 5.28 \text{ mH}
\]

\[
I(t) = \frac{V_{SW}(0^-) \sin(\omega_0 t)}{Z_0} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad Z_0 = \sqrt{\frac{L}{C}}
\]

Worst case, \( V_P = \frac{13800}{\sqrt{3}} = 11268 \text{ V}, \quad V_C = -10000 \text{ V}
\]

\[
(2) \quad I(t) = \frac{11268 - (-10000)}{25.1} \sin(4760t) = 847 \sin(4760t) \text{ A}.
\]

\[
V_C(t) = V_C(t) + \frac{1}{C} \int_0^t I(t) dt = V_C + \frac{V_{SW}(0^-)}{C Z_0} \int_0^t \sin(\omega_0 t) dt = V_C - \frac{V_{SW}(0^-)}{\omega_0 C Z_0} [\cos(\omega_0 t)]
\]

\[
V_C(+) = V_C + V_{SW}(0^-) [1 - \cos(\omega_0 t)]
\]

\[
(2) \quad V_C(+) = -10000 + 21268 [1 - \cos(4760t)]
\]

\[
d) \quad \alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}, \quad L = \frac{\alpha}{\omega_0}, \quad L = \frac{R V_L}{2L} = \frac{R}{2 \sqrt{C}} = \frac{R}{2 \zeta_0}
\]

\[
(2) \quad R = 2 \zeta_0 L = 2(25.1)(0.25) = 12.55 \Omega
\]
Problem 2. TRV creates high-frequency ringing voltages across circuit breakers. Consider the lossless situation where, on a 13.8kV three-phase system, a circuit breaker interrupts a 6000 Arms fault. The parasitic capacitance of the breaker and buswork for each phase is 0.1 μF.

a. Determine the time-domain expression for the TRV voltage across the breaker after it opens.

b. Determine the average volts-per-second rise for the first peak-to-peak swing of the transient.

c. If a 1000 Ω damping resistor is connected across the breaker for two cycles of 60Hz, compute the normalized damping ratio $\zeta$, and the Joule rating of the resistor.

\[ X_{sys} = \frac{V_{LN RMS}}{I_{sc RMS}} = \frac{13800\sqrt{3}}{6000} = 1.328 \Omega, \quad L_{sys} = \frac{X_{sys}}{\omega} = \frac{3.52 \text{ mH}}{120\pi} = 2.5 \text{ mH} \]

\[ t = 0^+ \quad V_P \left( \frac{d}{V_P} \right) + V_C = 0^+ \quad i(t) = \frac{V_P}{Z_0} \sin(\omega_0 t) \]

\[ V_C(t) = \int_{0}^{t} i(t) \, dt = V_P \left( 1 - \cos(\omega_0 t) \right), \quad V_P = 13800\sqrt{3} = 11268 \text{ V}, \quad \omega_0 = \frac{1}{\sqrt{L_C}} = 53300 \text{ rad/s} \]

\[ V_C(t) = 11268 \left( 1 - \cos(53300 t) \right) = V_{sw}(t) \]

(5)

b. First swing, $\Delta V = 22536$, $\Delta T = 58.9 \mu$s

\[ \frac{\Delta V}{\Delta T} = \frac{22536}{58.9 \mu s} = 383 \text{ V/μs} \]

(2)

c. The transient is insignificant in the 33.3 μs energy calculation.

\[ P_{avg} = \frac{V_{rms}^2}{R}, \quad W = P_{avg} \times T = \frac{(13800\sqrt{3})^2}{1000} \times (33.3 \mu s) \]

(3)

\[ \boxed{W = 2114 \text{ J}} \]
Problem 3. A circuit breaker interrupts the current to an unloaded transformer slightly after current zero. Actual voltages and currents at $t = 0^-$ are shown below. Starting with the general form $v(t) = V_F + e^{-\alpha t} [B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)]$, determine the time domain expression for the capacitor voltage. Determine numerical values for $V_F, \alpha, \omega_d, B_1, B_2$. As the last step, convert your expression to polar form with numerical values.

\[
\begin{align*}
\alpha &= \frac{1}{2RC} = 2500 \text{ N/s} \\
\omega_0 &= \frac{1}{\sqrt{LC}} = 5000 \text{ r/s}, \quad \xi = \frac{\alpha}{\omega_0} = 0.5 \\
\omega_d &= \omega_0 \sqrt{1 - \xi^2} = 4330 \text{ r/s} \\
V(0) &= B_1 = 10000 \text{ V} \\
\frac{dv}{dt} &= -\alpha e^{-\alpha t} [B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)] + \omega_d [w_1 \sin(\omega_d t) + w_2 \cos(\omega_d t)] \\
\frac{dv}{dt} &= -\alpha B_1 + \omega_d B_2, \quad B_2 = \frac{\frac{dv}{dt} + \alpha B_1}{\omega_d}
\end{align*}
\]

For the cap, $\dot{\lambda}_c = \frac{C \frac{dv}{dt}}{C}, \quad \frac{dv}{dt} = \frac{\dot{\lambda}_c}{C}$

\[
\begin{align*}
10000 &= 0.1A + 0.7A + 0.8A \\
\frac{dv}{dt} &= -0.8A = -400 \times 10^6 \text{ V/s} \\
B_2 &= -400 \times 10^6 + (2500)(10000) = -86605 \text{ V} \\
\theta &= -83.4^\circ \\
\varphi &= \frac{\arctan(B_2 \hat{B}_1)}{4330} = 87180 \text{ V cos}(4330t + 83.4^\circ) \\
\end{align*}
\]

Check $87180 \cos(83.4^\circ) = 10020 \text{ (OK)}$